

### 9.3: Systems of Nonlinear Equations and Inequalities

- **A system of nonlinear equations** is a system of two or more equations where at least one equation is nonlinear.
- **How to solve when one of the two equations is linear:** Solve for one variable in the linear equation and replace in the other equation. Then solve for the other variable.
- **Other methods to solve:** Eliminate one or more variables. Solve for remaining variables. Substitute and solve for the variables that were eliminated.
- **Nonlinear systems of inequalities:** Solve for intersections using the above methods. Graph the equations dividing the entire plane into two or more regions. Choose which regions fits the inequality.
- **Applications:** Aside from direct applications in solving models with inequalities, in calculus we often use the techniques to find area and volume of different regions.

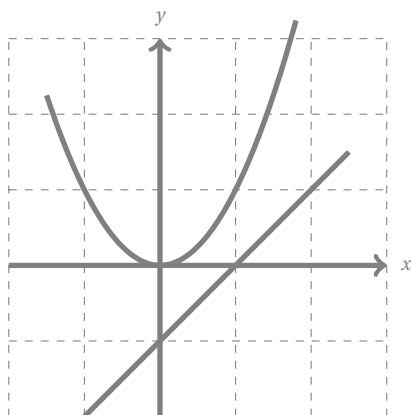
1. Solve each of the following systems of equations, then match each system with the corresponding graph. Mark the solutions on the corresponding graph.

(a)  $\begin{cases} y = x^2 - 1 \\ y = x + 1 \end{cases}$

(b)  $\begin{cases} y = x^2 - 1 \\ y = 2x - 2 \end{cases}$

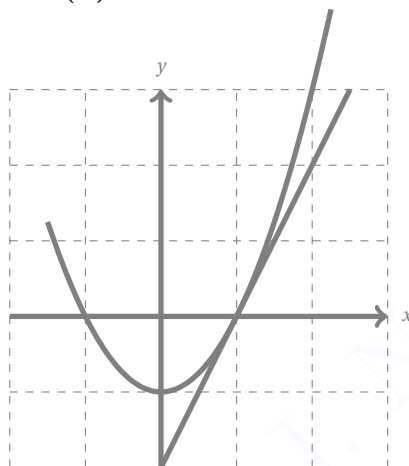
(c)  $\begin{cases} y = x^2 \\ y = x - 1 \end{cases}$

(i)



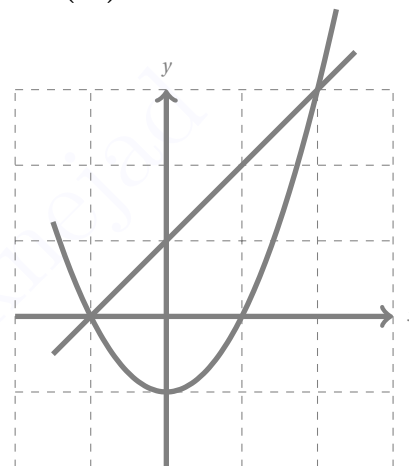
No solution.

(ii)



One solution.

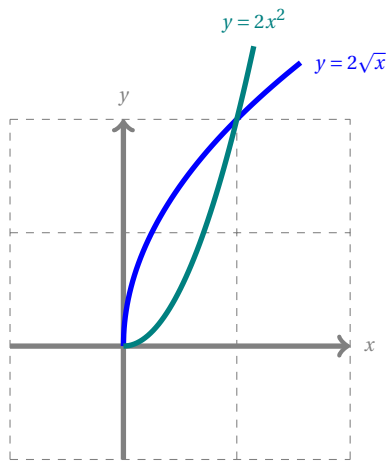
(iii)



Two solutions.

2. Solve the following system of equations and mark the solutions on the graph of two functions.

$$\begin{cases} y = 2x^2 \\ y = 2\sqrt{x} \end{cases}$$

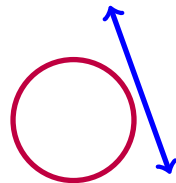


3. (a) Solve the following system.

$$\begin{cases} x^2 + y^2 = 16 \\ \frac{x}{3} + \frac{y}{2} = 1 \end{cases}$$

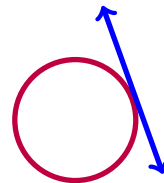
(b) Which of the following figures best describes your answer?

(i)



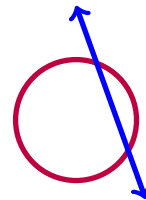
No solution.

(ii)



One solution.

(iii)

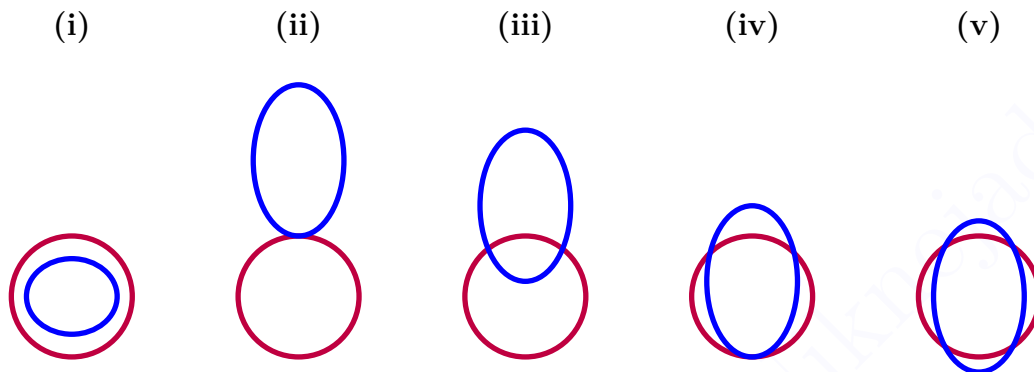


Two solutions.

4. (a) Solve the following system.

$$\begin{cases} x^2 + y^2 = 16 \\ \frac{x^2}{9} + \frac{y^2}{25} = 1 \end{cases}$$

(b) Which of the following figures best describes your answer?

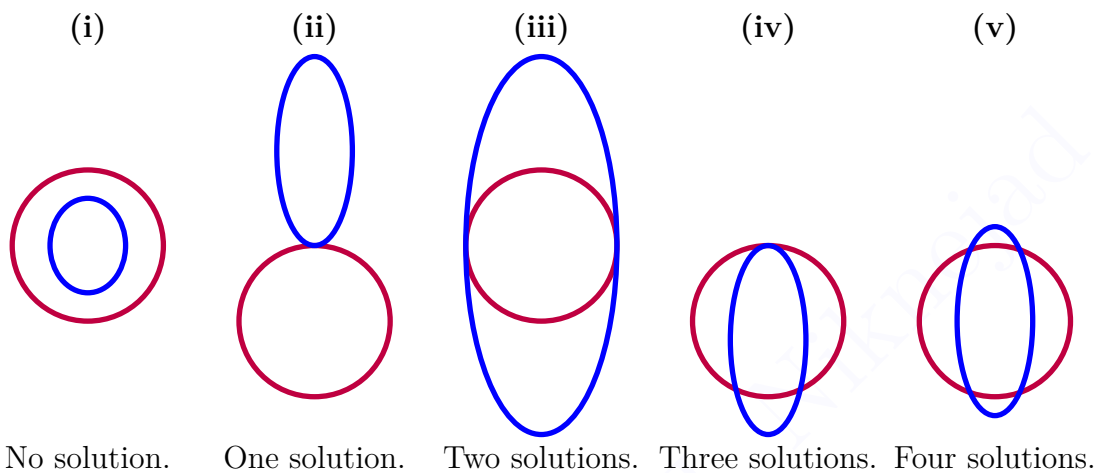


No solution.      One solution.      Two solutions.      Three solutions.      Four solutions.

5. (a) Solve the following system.

$$\begin{cases} x^2 + y^2 = 4 \\ \frac{x^2}{4} + \frac{y^2}{25} = 1 \end{cases}$$

(b) Which of the following figures best describes your answer?



6. (a) Solve the following system.

$$\begin{cases} x^2 + y^2 = 16 \\ \frac{(x-7)^2}{9} + \frac{y^2}{25} = 1 \end{cases}$$

(b) Which of the following figures best describes your answer?

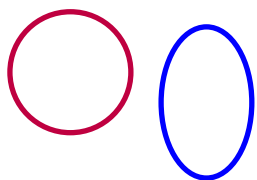
(i)

(ii)

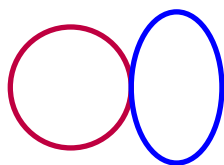
(iii)

(iv)

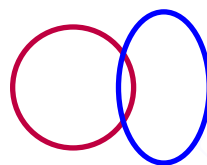
(v)



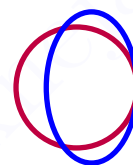
No solution.



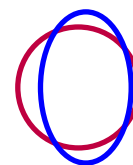
One solution.



Two solutions.

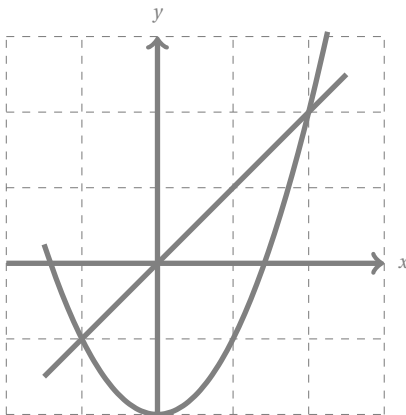


Three solutions.

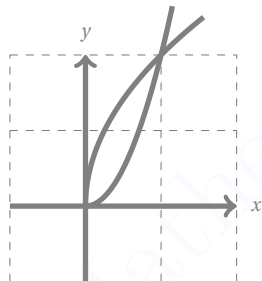


Four solutions.

7. To solve the system of inequalities  $\begin{cases} y \geq x^2 - 2 \\ y < x \end{cases}$ , we graphed two functions  $y = x^2 - 2$  and  $y = x$ . Shade the region of solutions to the system of inequalities noting dashed lines versus solid lines and points of intersections of the two graphs.



8. To solve the system of inequalities  $\begin{cases} y > 2x^2 \\ y \leq 2\sqrt{x} \end{cases}$ , we graphed two functions  $y = 2x^2$  and  $y = 2\sqrt{x}$ . Shade the region of solutions to the system of inequalities noting dashed lines versus solid lines and points of intersections of the two graphs.



9. A car braked with a constant deceleration of  $16 \text{ ft/s}^2$ , producing skid marks measuring 200 feet before coming to a stop. The velocity function for the car is  $v(t) = -16t + v_0$  and the position function is  $s(t) = -8t^2 + v_0t$ , where  $v_0$  is the initial velocity when the brakes were applied. How fast was the car traveling when the brakes were first applied? (That is, find  $v_0$ .)

*Hints: You need to solve a non-linear system of equations in two variables to solve for  $v_0$  and  $T_f$ , where  $T_f$  is the time that it takes for the car to stop.*

## Related Videos:

1. **Example 1:** [https://mediahub.ku.edu/media/MATH+-+System+of+Non-linear+Equations+1/1\\_09vwube5](https://mediahub.ku.edu/media/MATH+-+System+of+Non-linear+Equations+1/1_09vwube5)
2. **Example 2:** [https://mediahub.ku.edu/media/MATH+-+System+of+Non-linear+Equations+2.m4v/1\\_y6gicag](https://mediahub.ku.edu/media/MATH+-+System+of+Non-linear+Equations+2.m4v/1_y6gicag)

KU Mathematics – J. Niknejad